1 Quantum Mechanics

SYLLABUS

Quantum mechanics- Failure of classical physics to explain black body spectra, Planck's radiation law, Compton Effect, Wave particle duality, de Broglie's hypothesis, Concept of wave and group velocity, Experimental demonstration of matter waves, Davisson and Germer experiment, Heisenberg's uncertainty principle and Thought experiment.

INTRODUCTION

Prof. Max Planck proposed the quantum theory of radiation to explain the distribution of energy with wavelength in black body radiation.

Quantum theory of radiation has been able to explain a number of phenomenons concerning the interaction of energy with matter. For example, it gives very satisfactory explanation of variation of specific heat of solids with temperature, Photo-electric effect, Compton Effect etc. This theory was given by Planck in his historic paper titled "Theory of Law of distribution of energy in a Normal spectrum" presented before the Berlin Academy of Sciences on Dec-14, 1900.

Q. 1. What are black body spectra? Give the failure of classical physics to explain Black body spectra.

Ans: A black body is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. A black body in thermal equilibrium emits electromagnetic radiation called black body radiation and the related spectrum is known as Black body spectrum.

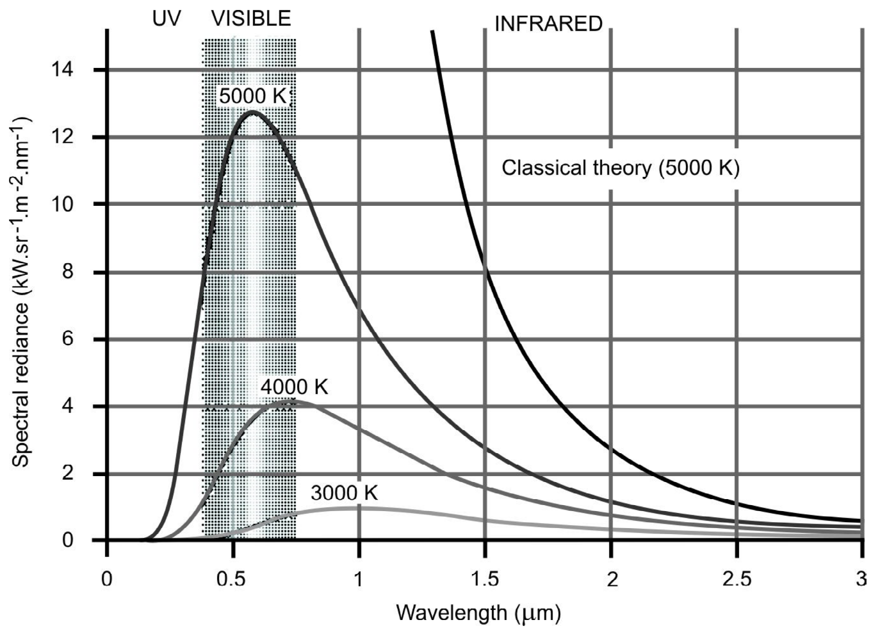

Fig. 1.1 Black Body radiation curve (Observed & Classically Predicted)

In physics, the Rayleigh-Jeans law attempts to describe the spectral radiance of electromagnetic radiation at all wavelengths from a black body at a given temperature through classical arguments.

The Rayleigh-Jeans law agrees with experimental results at large wavelengths (low frequencies) but strongly disagrees at short wavelengths (high frequencies), depicted as "Classical Theory" in the graph. This inconsistency between observations and the predictions of Classical Physics is commonly known as the ultraviolet catastrophe which is difference between Black curve (as classically depicted by Rayleigh-Jeans law) and the Blue curve (The measured curve as predicted by Planck's law). Its resolution was a foundational aspect of the development of Quantum Mechanics in the early 20^{th} century.

Planck's Radiation Law

Q. 2. Write a short note on Quantum theory of radiation

Ans: In 1900, Prof. Max Planck proposed the quantum theory of radiation to explain the distribution of energy with wavelength in black body radiation.

According to Plancks Hypothesis

1) A black body radiation chamber filled up with standing

waves equivalent to simple harmonic oscillators or resonators of molecular dimensions, known as Plancks oscillators which can vibrate with all possible frequencies.

2) The oscillator emits energy only when it passes from a higher energy state to a lower energy state and absorbs energy only when it passes from a lower energy state to a higher energy state. * No emission or absorption of energy takes place when the oscillator is in a given state.

3) The oscillators in the cavity walls limited to energies of

$$E_n = nh\nu$$

Where, í is frequency of oscillator

h is Plancks constant = 6.62×10^{-34} Joules sec

n = 0, 1 , 2, 3..........

Note : Planck's radiation Law–

Mathematically Planck's radiation formula for the energy $u(\lambda)$ radiated per unit volume by a cavity of a blackbody in the wavelength interval $(\lambda + d\lambda)$ in terms of wavelength is given by

Q. 3. Derive the Plancks radiation law.

Ans: According to Plancks Hypothesis

1) A black body radiation chamber filled up with standing waves equivalent to simple harmonic oscillators or resonators of molecular dimensions, known as Plancks oscillators which can vibrate with all possible frequencies.

2) * The oscillator emits energy only when it passes from a higher energy state to a lower energy state and absorbs energy only when it passes from a lower energy state to a higher energy state. * No emission or absorption of energy takes place when the oscillator is in a given state.

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$$E_n = nh\nu$$

Where, í is frequency of oscillator

h is Plancks constant = 6.62×10^{-34} Joules sec

n = 0, 1 , 2, 3..........

The average number of photons f(í) in each state of energy E = hv by Bose Einstein distribution function is given by by Bose Einstein distribution function is given by

$$f(\nu) = \frac{1}{e^{\frac{h\nu}{kt}} - 1} \qquad \text{..........(1)}$$

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \qquad \text{..........(2)}$$

Therefore the energy density of photons in a cavity is accordingly

$$u(\nu)d\nu = \text{Avg. energy of oscillator} \times \text{density of oscillator}$$

$$= h\nu G(\nu)f(\nu)d\nu \qquad \text{..........(3)}$$

$$\therefore \quad u(\nu)d\nu = \frac{8\pi h}{c^3} \cdot \frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}} - 1} \qquad \text{..........(4)}$$

Equation (4) represents the Planck's radiation formula in terms of frequency.

Further $\nu = \frac{c}{\lambda}$

$$\therefore \quad u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} d\lambda \qquad \text{..........(5)}$$

Equation (5) represents the Planck's radiation formula in terms of wavelength.

Q. 4. What is Photon? State its properties.***

Ans: According to Planck's quantum hypothesis, a radiation of frequency í is emitted as quantum of energy hí.

Where, h is Planck's Universal constant = 6.62×10^{-34} Joules sec

The quantum is the basic unit of light energy & cannot be subdivided. It is the atom of energy and is known as photon.

Properties:

A photon of light of frequency í has the momentum

$$p = \frac{h\nu}{c}$$

$$= \frac{h}{\lambda} \qquad \because \quad \nu = \frac{c}{\lambda}$$

A photon of light of frequency í has the energy

$$E = h\nu$$

$$= \frac{hc}{\lambda} \qquad \because \quad \nu = \frac{c}{\lambda}$$

A photon has zero rest mass, zero charge and spin equal to one quantum unit i.e. $\hbar \left(\hbar = \frac{h}{2\pi} \right)$

COMPTON EFFECT

Q. 5. What is Compton Effect? Derive an expression for Compton shift and wavelength of scattered photon. ***

Ans: **Compton Effect :** When X-rays falls on matter, a part is scattered without any change in wavelength. This is known as Unmodified or coherent or classical scattering. In addition to classical scattering the secondary radiation contains X-rays of lower frequency or higher wavelength than those of the incident beam. This is known as modified or incoherent scattering. The phenomenon is known as Compton effect. The scattered electron is known as Compton recoil electron.

Compton assumption:

The X-rays of frequency ν (wavelength λ) consists of photons of energy $h\nu$ and momentum $\frac{h}{\lambda} = \frac{h\nu}{c}$.

1. The electrons are assumed to be free and stationary.
2. The collision between the high energy photon and electron is elastic.

Compton Shift:

Suppose an X-ray photon of frequency ν and energy $h\nu$ collides with an electron at rest. A part of its energy is imparted to the electron which is ejected with a velocity v in a direction making an angle Φ with that of the incident X-ray photon. The remaining energy is given out as a scattered X-ray photon of lower frequency ν', moving in the direction making an angle Φ with that of the incident photon.

If m_0 is the rest mass of the electron, then its mass while moving with a velocity v according to the theory of relativity is given by $m = \frac{m}{\sqrt{1 - \frac{\nu^2}{c^2}}}$ where, c is the velocity of light.

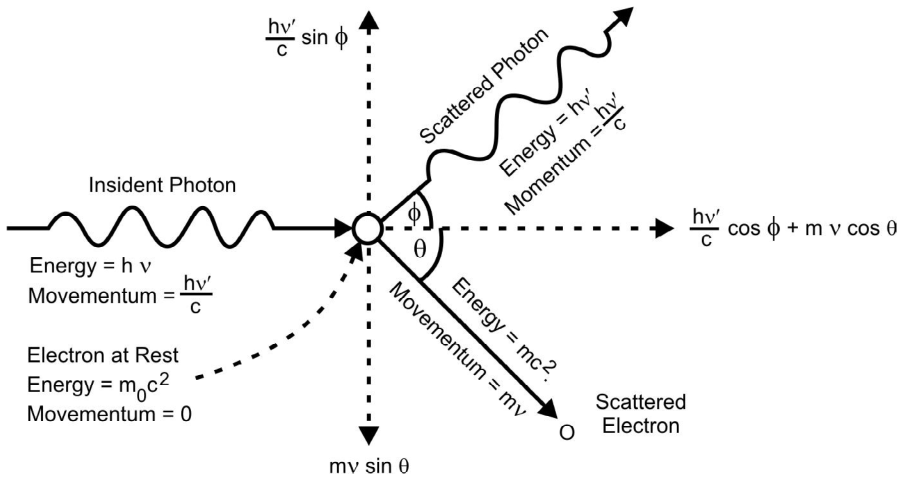

Fig. 1.2

(a) Applying the principle of conservation of energy, we have

$$h\nu + m_0c^2 = h\nu' + mc^2$$

or $mc^2 = h(\nu - \nu') + m_0c^2$

Squaring, we get

$$m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')m_0c^2 + m_0{}^2c^4 \qquad \text{..........(1)}$$

(b) Applying the principle of conservation of momentum, we have Along X-axis

Momentum of Photon before collision + Momentum of Electron before collision =

Momentum of scattered photon + Momentum of scattered electron

Therefore $\dfrac{h\nu}{c} + 00 = \dfrac{h\nu'}{c}\cos\phi + mV\cos\theta$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + mv\cos\theta$$

or $mv\cos\theta = h(\nu - \nu'\cos\phi)$ ....(2)

Along Y-axis

$$0 = \frac{h\nu'}{c}\sin\phi - mv\sin\theta$$

or $mvc\sin\theta = h\nu'\sin\phi$

$$m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\phi + \nu'^2\cos^2\phi + \nu'^2\sin^2\phi)$$

$$m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\phi + \nu'^2)$$

Subtracting (4) from (1), we get

$$m^2c^2(c^2 - v^2) = -2\nu\nu'h^2(1-\cos\phi) + 2h(\nu-\nu')m_0c^2 + m_0^{\ 2}c^4$$

or $$\frac{m_0^{\ 2}c^2}{\left(1-\dfrac{v^2}{c^2}\right)}(c^2 - v^2) = -2\nu\nu'h^2(1-\cos\phi) + 2h(\nu-\nu')m_0c^2 + m_0^{\ 2}c^4$$

or $$2\nu\nu'h^2(1-\cos\phi) = 2h(\nu-\nu')m_0c^2$$

or $$\frac{\nu-\nu'}{\nu\nu'} = \frac{h(1-\cos\phi)}{m_0c^2}$$

or $$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\phi)$$

or $$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\phi)$$

If λ and λ' are the wavelengths corresponding to the frequencies ν and ν' respectively, then

Compton shift $\lambda' - \lambda = \Delta\lambda = \dfrac{h}{m_0c}(1-\cos\phi)$

This relation is known as Compton equation and shows that the increase in wavelength or Compton shift $\Delta\lambda$ is independent of the wavelength of the incident radiation as well as the nature of the scattering substance, but depends upon the angle of scattering.

The wavelength of the scattered photon is obtained as

$$\lambda' = \lambda + \Delta\lambda = \lambda + \frac{h}{m_0c}(1-\cos\phi)$$

Q. 6. What is Compton wavelength? Determine its value. Distinguish between Compton shift and Compton wavelength.

Ans: Compton wavelength: Compton shift for an angle of scattering $\Phi = 90^\circ$ is called Compton wavelength. It is denoted as λ_c.

We know,

Compton shift $\Delta\lambda = \dfrac{h}{m_0c}(1-\cos\phi)$ and $\lambda_c = \Delta\lambda$ at $\Phi = 90^\circ$

Therefore, $\lambda_c = \dfrac{h}{m_0 c}(1 - \cos 90°)$

$$= \frac{h}{m_0 c} = \frac{6.624 \times 10^{-34}}{9.107 \times 10^{-31} \times 2.998 \times 10^{8}}$$

$$= 0.0242 \times 10^{-10}\,\text{m} = 0.0242\,\text{A}°$$

Distinction between Compton shift and Compton wavelength: Compton wavelength is a constant quantity equal to 0.0242 A^0. It is the Compton shift for an angle of scattering $\Phi = 90°$. Compton shift is a variable quantity depending upon the angle of scattering.

Compton shift $\Delta\lambda = \dfrac{h}{m_0 c}(1 - \cos\phi)$ and

Compton Wavelength $\lambda_c = \dfrac{h}{m_0 c}(1 - \cos 90°) = \dfrac{h}{m_0 c}$

Therefore, $\Delta\lambda = \lambda_c(1 - \cos\phi)$

As the Compton shift is proportional to $(1 - \cos\Phi)$, it increases from zero (for $\Phi = 0$) to $\Delta\lambda = 2\lambda_c$ (for $\Phi = \pi$).

Q. 7. Discuss the significance of Compton effect.

Ans: The Compton effect clearly shows the particle like nature of electromagnetic radiation. Not only a precise quantum of energy hí can be assigned to a photon but also a precise quantum of momentum $h\nu / c = h / \lambda$. The total momentum of a monochromatic radiation cannot have any value but is only an exact multiple of the linear momentum of a single photon. In other words the momentum as well as the energy of the electromagnetic radiation is quantized. The phenomenon of quantum effect is thus due to the elastic collision between two particles, the photon of the incident radiation and electron of the scatterer.

Q. 8. Why Compton effect cannot be observed with visible light?

Ans: The energy of a visible light photon, say of wavelength λ = 6000 × 10^{-10}m is given by

$$h\nu = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{6000 \times 10^{-10} \times 1.6 \times 10^{-19}} = 2\text{eV(Approx.)}$$

Whereas, the energy of an X-ray photon, say of wavelength 1A° will be more than 10^3 times the above value. The energy of an ultra violet photon is of the order of the 10 eV.

The B.E. of the electron in the atom is of the order of 10 eV. Also The B.E. of the electron in the hydrogen atom is of the order of 13.6 eV.

Hence these electrons can be treated as free when X-rays or gamma rays are incident but these cannot be treated as free for visible or ultra violet light. Thus for visible or ultra violet light the whole of the atom takes part in quantum scattering and not the electron alone.

The Compton wavelength for the electron $\lambda_c = \frac{h}{m_0 c} = 0.024\,\text{A}°$

and for an atom of carbon is 1.1×10^{-6} A°. To detect a change in wavelength of this order, the optical instrument should have a resolving power.

$$\frac{\lambda}{\Delta\lambda} = \frac{6000}{1.1 \times 10^{-6}} = 5 \times 10^9$$

which is much beyond the limit of resolution of these instruments. Hence Compton effect cannot be observed with visible or ultra violet light.

Q. 9. Explain wave particle duality of light. Or Give an account of duel nature of light.

Ans: It is well known that, light exhibit the phenomenon of interference, diffraction, polarization, photo-electric effect, Compton Effect and discrete emission and absorption.

The phenomena of interference, diffraction and polarization can only be explained on the basis of wave theory of light. These phenomena show that light possesses wave nature. On the other hand the phenomena of photo-electric effect, Compton Effect and discrete emission and absorption can only be explained on the basis of quantum theory of light, according to which light is propagated in small packets or bundles of energy hí, where í is the frequency of radiation. These packets are called photons or quanta and behave like Corpuscles. Thus these latter phenomena indicate that light possesses particle (corpuscular) nature.

Thus we can say that light possesses duel nature. In some experiments it behaves as wave while in other experiments it behaves as particles.

de Broglie Hypothesis

Q. 10. Give an account of wave nature of matter.

Ans: In 1923-24, de Broglie proposed that the idea of duel nature (i. e. wave-particle duality) should be extended to all micro particles, associating both wave and corpuscular characteristics with every particle. The experiments such as those in which e/m of the material particles (e.g. electron, α-particle etc.) is measured, indicate that the matter possesses particle nature.

But in 1923 Louis de Broglie proposed that material particle such as an electron, proton etc., might have a duel nature, just as light does. According to de Broglie a moving particle, whatever its nature, has wave properties associated with it. de Broglie proposed that the wavelength λ associated with any moving particle of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \text{ where h is Planck's constant}$$

Note: Remember the concept.

Fig. 1.3

Energy of photon as particle   Energy of Photon as Wave

$mc2 = h\nu$

mass, $m = h\nu/c2$ ....(1)

and momentum, $p = mc = h\nu/c$ ....(2)

Therefore Lambada = h/p

This is de Broglie wave equation

Q. 11. State the de Broglie hypothesis of matter waves and obtain an expression for de Broglie wavelength for matter waves in terms of K.E. and Temperature.***

Ans:

de Broglie hypothesis:

According to de Broglie a moving particle, whatever its nature, has wave properties associated with it. Louis de Broglie proposed that the wavelength ë associated with any moving particle of momentum p (momentum p = mass m x velocity v) is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{..........(1)}$$

where h is Planck's constant

This is known as de Broglie wave equation and λ is called de Broglie wavelength.

(a) de Broglie wavelength in terms of energy:

If E is the K.E. of the moving particle, then

$$E = \frac{1}{2}mv^2 = \frac{1m^2v^2}{2m} = \frac{p^2}{2m} \text{ gives } p = \sqrt{2mE}$$

put in equation (1)

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad \text{..........(2)}$$

(b) de Broglie wavelength in terms of temperature:

Acc. to Kinetic theory of gases, the average K.E. of material particle is given by

$E = \frac{3}{2}kT$ Where, k is Boltzmann's constant = 1.38×10^{-23} JK-1 and T is absolute Temperature.

Put in equation (1)

$$\lambda = \frac{h}{\sqrt{2mkT}} \qquad \text{..........(3)}$$

Equation (2) & (3) gives de Broglie wavelength for matter waves in terms of K.E. and Temperature respectively.

Note: (c) de Broglie wavelength for accelerated charged particle–

If a charged particle carrying charge q is accelerated through a potential difference V volts, then Kinetic Energy E = qv

Put in equation (1)

$$\lambda = \frac{h}{\sqrt{2mqv}} \qquad \text{..........(4)}$$

Phase velocity & Group velocity

Q. 12. Define de-Broglie phase velocity (wave velocity) and show that it exceeds velocity of light as well as the particle velocity.

Ans: **Phase or Wave velocity:** The velocity with which the plane of equal phase of progressive wave travels through the medium is known as phase velocity (wave velocity).

If λ is wavelength and í is the frequency of the wave, then

$$\text{Phase velocity } v_p = \nu\lambda \qquad \text{..........(1)}$$

According to quantum theory, the energy of the wave

$$E = h\nu \qquad \therefore \nu = \frac{E}{h}$$

and momentum $p = \frac{h}{\lambda} \qquad \therefore \lambda = \frac{h}{p}$

Substituting in (1), we have, the phase velocity

$$v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p} = \qquad \text{..........(2)}$$

This relation holds true in the case of de-Broglie waves associated with a moving particle.

de-Broglie phase velocity exceeds velocity of light as well as the particle velocity : In a non-relativistic case for a particle of mass m, moving with velocity v, the total energy E = mc^2 and momentum p = m. v.

Therefore Phase velocity of the associated de-Broglie wave

$$v_p = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v} \qquad \text{..........(3)}$$

As v < c the phase velocity of the associated wave is always greater than c, the velocity of light in vacuum.

As a material particle can never have even the velocity of light, the phase velocity also exceeds the particle velocity.

Q. 13. Define Group velocity. Show that the particle velocity is equal to the Group velocity of the de Broglie waves.***

Ans: **Group velocity:** The velocity with which the slowly varying envelop of the modulated pattern due to a group of waves travels in a medium is known as the Group velocity. Thus a wave packet (comprises a group of waves each with slightly different velocity

& wavelength) moving with the velocity $v_g = \frac{d\omega}{dk}$ is called the Group velocity.

According to de-Broglie's hypothesis, wavelength associated with a particle of momentum p is given by, $\lambda = \frac{h}{p}$

$$\therefore \quad = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k \qquad \text{Since,} \quad \frac{h}{2\pi} = \hbar$$

$$\therefore \quad dk = \frac{dp}{\hbar} \qquad \text{..........(1)}$$

de-Broglie assumed the Planck's relation, E=hv true for radiations as well as particle.

$$\therefore \quad E = h\nu = \frac{h}{2\pi} 2\pi\nu = \hbar\omega$$

$$\therefore \quad d\omega = \frac{dE}{\hbar} \qquad \text{..........(2)}$$

Hence, group velocity for de-Broglie waves is

$$V_g = \frac{d\omega}{dk} = \frac{dE/\cancel{\hbar}}{dp/\cancel{\hbar}} = \frac{dE}{dp} \qquad \text{..........(3)}$$

But, the relativistic energy is given by

$$E^2 = p^2c^2 + m_0^2c^4$$

Differentiating above equation with respect to p, we get

$$2E\frac{dE}{dp} = 2pc^2$$

$$\therefore \quad \frac{dE}{dp} = \frac{pc^2}{E} \qquad \text{..........(4)}$$

But, E= mc2 and p = m v   Where, v is particle velocity.

Hence eq. (4) becomes

$$\frac{dE}{dp} = V \qquad \text{..........(5)}$$

Comparing eqs. (3) & (5), we get

$$\boxed{V_g = V}$$

Thus, the de-Broglie wave group associated with a moving particle travels with the same velocity as the particle.

Q. 14. Define Phase velocity and Group velocity. Derive a relation between Group velocity and Phase velocity.

Ans: **Phase or Wave velocity :** The velocity with which the plane of equal phase of progressive wave travels through the medium is known as phase velocity (wave velocity).

If λ is wavelength and í is the frequency of the wave, then

Phase velocity $v_p = \nu\lambda$

..........(1)

According to quantum theory, the energy of the wave

$$E = h\nu \qquad \therefore \nu = \frac{E}{h}$$

and momentum

Substituting in (1), we have, the phase velocity

$$v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p} = \frac{\hbar\omega}{\hbar k} = \frac{\omega}{k}$$

..........(2)

This relation holds true in the case of de-Broglie waves associated with a moving particle.

Group velocity : The velocity with which the slowly varying envelop of the modulated pattern due to a group of waves travels in a medium is known as the Group velocity. Thus a wave packet (comprises a group of waves each with slightly different velocity & wavelength) moving with the velocity $v_g = \frac{d\omega}{dk}$ is called the Group velocity.

Relation between Phase velocity and Group velocity–

We have the relation for phase velocity,

$$V_p = \frac{\omega}{k} \qquad \therefore \quad \omega = V_p.k$$

$$\therefore \quad \frac{d\omega}{dk} = V_p + k\frac{dV_p}{dk}$$

But, $V_g = \frac{d\omega}{dk}$ $\therefore$ $V_g = V_p + k\frac{dV_p}{dk} = V_p + k\frac{dV_p}{d\lambda}.\frac{d\lambda}{dk}$ ....(3)

Since, $k = \frac{2\pi}{\lambda}$ $\therefore$ $\lambda = \frac{2\pi}{k}$ $\therefore$ $\frac{d\lambda}{dk} = -\frac{2\pi}{k^2}$

Substituting this value in eq. (1) we get

$$V_g = V_p - k\frac{dV_p}{d\lambda}.\left(\frac{2\pi}{k^2}\right)$$

$$\therefore \quad V_g = V_p - \frac{dV_p}{d\lambda}.\left(\frac{2\pi}{k}\right) \qquad \text{but,} \quad \therefore \quad \lambda = \frac{2\pi}{k}$$

$$\therefore \quad V_g = V_p - \lambda\frac{dV_p}{d\lambda} \qquad ....(4)$$

This is the relation between Vg and Vp.

Experimental Demonstration of Matter waves

Q. 15. Give the examples of experimental demonstration of matter waves.

Ans: Following experiments give demonstration of matter waves i.e. material particle shows wave like properties.

1) Davisson and Germer experiment- demonstrates the diffraction of electron, the first experimental evidence.
2) G. P. Thomson experiment- famous electron diffraction experiment.

Davisson & Germer Experiment

Q.16. Describe Davisson and Germer's experiment for study of electron diffraction. How this experiment confirms wave nature of electron beam?

Ans: The first experimental demonstration of matter waves i.e. electron diffraction is given by Davisson and Germer.

- **Experimental Arrangement:**

  It consists of Electron Gun, Target and Detector. (As shown in figure below)

  Electron Gun : The electrons are produced by heating a tungsten filament F and are accelerated through a known potential difference V by maintaining a constant potential difference between the filament F and a plate P.

Target : It is a single crystal of Nickel.

Detector : It is an ionization chamber connected to a sensitive galvanometer.

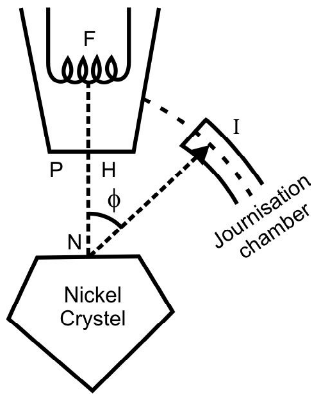

Fig. 1.4

- **Experimental Procedure:** The fine beam of electrons comes from electron gun and falls normally on the surface of a single Nickel crystal. The electrons are scattered by the crystal in all directions and the intensity of scattered beam in any direction is measured by Detector.

  The results of the experiment are represented in the form of a polar graph. (In the polar graph, a line is drawn from the origin O inclined at an angle Φ with the direction of incident beam. The length of the line drawn from O is taken proportional to the intensity of the diffracted electron beam. The end points of these lines drawn for different values of Φ are joined together to get smooth, continuous curve. This curve is known as the Polar graph.)

- **Result :** To study the effect of increasing electron energy on the scattering angle Φ, the observations were taken for increasing values of accelerating voltages.

  At a P.D. of 44 V, a spur or hump appears at $\Phi = 60°$.

  At a P.D. of 54 V, a spur or hump appears at $\Phi = 50°$.

  At a P.D. of 68 V, a spur or hump appears at $\Phi = 40°$, as shown below.

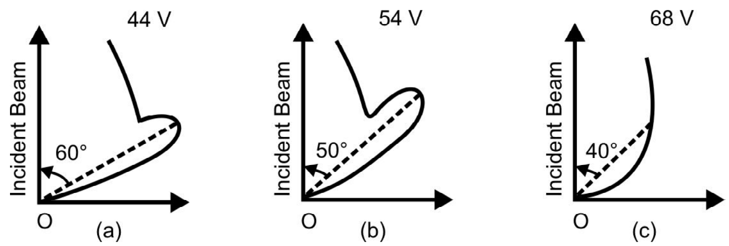

Fig. 1.5

Intensity against scattering angle for 54 volt, Shows electrons with K.E. 54 eV suffer maximum scattering at $\Phi = 50°$.

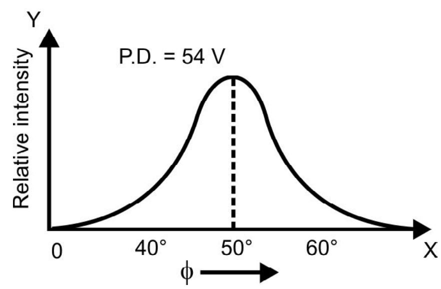

Fig. 1.6

- **Theoretical Explanation:**

  The occurrence of spur at $\Phi = 50°$ with the electrons accelerated through 54 V can be explained as –

  According to de-Broglie's theory, the wavelength associated with the electron accelerated through a P.D. of 54 Volts is given by

  $$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.26}{\sqrt{V}} \text{ in A°} = \frac{12.26}{\sqrt{54}}.\text{A°} = 1.66 \text{ A°} \qquad \text{..........(1)}$$

  Now, the wavelength calculated from diffraction experiment-

  According to Bragg's law, $2d.\sin\theta = n\lambda$.

  By putting 2d = 2.15 A^0 (since lattice constant of Nickel = 2.15 A°), θ = 50° and for first order maximum n = 1 , we get

  $$2.15\text{A°} \times \sin 50° = \lambda$$

  $$\therefore \quad \lambda = 2.15 \times 0.766 = 1.65 \text{ A°} \qquad \text{..........(1)}$$

Thus

Theoretical value of λ *from* de-Broglie's hypothesis =*Experimental value of ë from electron diffraction experiment*

Hence, this experiment confirms the wave nature of electron beam.

Heisenberg's uncertainty principle

Q. 17. State and explain Heisenberg's uncertainty principle.***

Ans: **Statement of Uncertainty Principle**:- Heisenberg's uncertainty principle states that *product of the uncertainties in determining the position and momentum of the particle is approximately equal to a number of the order of* $\hbar$, *where* $\hbar = h/2\pi$, *h being Planck's constant i.e.,*

$$\Delta p.\Delta x \approx \hbar \qquad \text{..........(a)}$$

Where Δp is the uncertainty in determining the momentum and Δx is the uncertainty in determining the position of the particle. (Say electron)

Thus the product of the uncertainties in determining the position and momentum of the particle can never be smaller than the number of the order $\frac{1}{2}\hbar$.

$$\therefore \quad \Delta p.\Delta x \geq \frac{1}{2}\hbar \qquad \text{..........(b)}$$

According to above relation the smaller is the value of Δx, i.e. more exactly we can determine the position, the larger is the value of Δp i.e. less exactly we can determine the momentum and vice versa.

The relation is universal and holds for all the canonical conjugate physical quantities like position and momentum, energy and time, angular momentum and angle, etc. whose product has dimensions of action (joule-sec).

Thus,

1) **Uncertainty relation for position and momentum:**

   Suppose p is the linear momentum associated with position x. Then,

$$\Delta p.\Delta x \geq \frac{1}{2}\hbar \qquad \text{..........(1)}$$

2) **Uncertainty relation for energy and time:**

   Suppose E is the Energy associated with time t. Then,

$$\Delta E.\Delta t \geq \frac{1}{2}\hbar \qquad \text{..........(2)}$$

3) **Uncertainty relation for angular momentum and angle:**

Suppose J is the angular momentum associated with angle θ. Then,

$$\Delta J.\Delta\theta \geq \frac{1}{2}\hbar \qquad \text{..........(3)}$$

Extra Note:

4) Uncertainty relation for number of photons and phase:

Suppose N is the number of photons associated with phase Φ. Then,

$$\Delta N.\Delta\phi \geq \frac{1}{2}$$

$$\because \quad \Delta E = \hbar\omega\Delta N \rightarrow \hbar\omega\Delta N\Delta t \geq \frac{\hbar}{2} \rightarrow \Delta N\Delta\omega t \geq \frac{1}{2}$$

Q. 18. Explain the validity of Heisenberg uncertainty principle by giving examples of Diffraction of a beam of electrons by a slit.

Ans: The validity of Heisenberg's uncertainty principle is illustrated very well by the following hypothetical experiment.

Diffraction of a beam of electrons by a slit:

A narrow beam of electron fall on the slit and after passing through the slit, spread out due to diffraction effect, so that the diffraction pattern is viewed on the screen as shown in figure below.

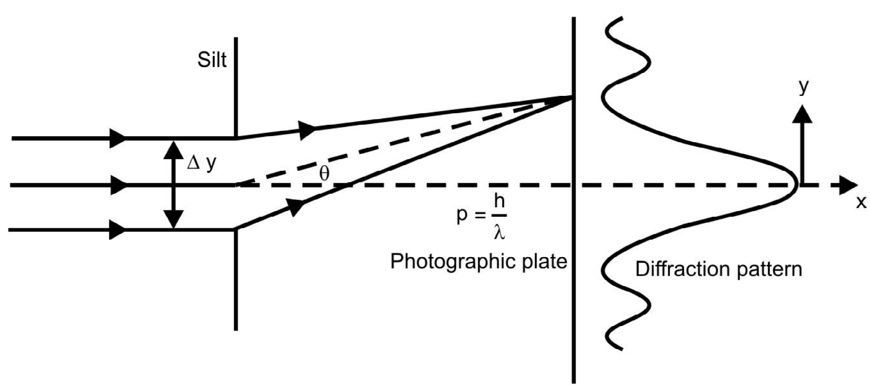

Fig. 1.7

By the diffraction due to single slit equation $d\sin\theta = n\lambda$, the minimum uncertainty in determining the position of electron along Y-axis is given by

$$\Delta y = \frac{\lambda}{\sin\theta} \qquad \text{..........(1)}$$

Where, λ = wavelength of electron,

θ = Angle of deviation corresponding to first minimum

Initially the electrons are moving along X-axis and so they had no component of momentum along Y-axis. But, the electrons deviated at the slit from their initial path to form the pattern. They acquire an additional component of momentum along Y-axis. If p is the momentum of the electron, the component of momentum of electron along Y-axis is p.sinè. As the electron may be anywhere within the pattern from angle $-\theta$ to $+\theta$, the Y-component of momentum of the electron may be anywhere between –p.sinè and p.sinè, so that the uncertainty is the Y-component of momentum of the electron,

$$\Delta p_y = p\sin\theta - (-p\sin\theta), \text{ i.e.,}$$

$$\Delta p_y = 2p\sin\theta$$

$$= 2\frac{h}{\lambda}\sin\theta \qquad \text{(since } \lambda = h/p\text{)} \qquad \text{..........(2)}$$

Multiplying (1) and (2), we get

$$\Delta y.\Delta p_y = \frac{\lambda}{\sin\theta}.\frac{2h}{\lambda}\sin\theta$$

$$= 2h$$

$$\text{or} \quad \Delta y.\Delta p_y \geq \frac{\hbar}{2} \qquad \text{..........(3)}$$

i.e., the product of uncertainty in determining the position and that in determining the momentum is always greater than $\hbar/2$; which is Heisenberg uncertainty principle.

Que.19- Explain why electron cannot exist inside a nucleus.

Ans: The radius of the nucleus of any atom is of the order of 10^{-14} m, so that if an electron is confined within nucleus, the uncertainty in its position must be greater than 10^{-14} m.

According to uncertainty principle

$$\Delta p.\Delta x \approx \hbar \qquad \text{..........(1)}$$

Where, Δx is the uncertainty in the position = 2r = 2 × 10^{-14} m

Δp is the uncertainty in the momentum and

$$\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ joule-sec}$$

Therefore equation (1) gives,

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-14}} = 5.275 \times 10^{-21} \text{kg.m/sec}$$

If this is the uncertainty in momentum of the electron, the momentum of the electron must be at least comparable with its magnitude, i.e.,

$$p = 5.275 \times 10^{-21} \text{kg.m/sec}$$

The K.E. of the electron of mass m is given by

$$T = \frac{p^2}{2m} \approx \frac{\left(5.275 \times 10^{-21}\right)^2}{2 \times 9 \times 10^{-31}} \text{ Joule} \quad (\text{since } m = 9 \times 10^{-31} \text{ kg})$$

$$\approx \frac{\left(5.275 \times 10^{-21}\right)^2}{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19}} \text{eV} \approx 9.7 \times 10^7 \text{eV} \approx 97 \text{MeV}$$

This means that if the electrons exist inside the nucleus, the K.E. must be of the order of 97 MeV. But experimental observations show that no electron in atom possess energy greater than 4 MeV. Clearly the inference is that the *electrons do not exist in the nucleus.*

Thought Experiment

Q. 20. Describe Heisenberg's gamma ray microscope – Thought experiment to prove Heisenberg's uncertainty principle.

OR

Derive Heisenberg uncertainty principle from a hypothetical gamma ray microscope.

Ans: **Thought experiment i.e. Heisenberg's gamma ray microscope:**

This method was suggested by Bohr and fully developed by Heisenberg.

Let us try to observe the position of an electron with a microscope of very high resolving power.

According to optical theory, the resolving power of microscope i.e. the distance between two points that can be just distinguished as separate by the microscope is given by

$$\Delta x = \frac{\lambda}{2\mathrm{Sin}\theta} \qquad ....(1)$$

Where, Δx = the distance between two points that can be just distinguished as separate by the microscope

λ = wavelength of light (gamma rays)

θ = semi-vertical angle

Now, in order to see an electron through the microscope, it is illuminated by gamma ray photons of energy hí.

A gamma ray photon after colliding with the electron at rest enters into the microscope so that the electron becomes visible as shown in figure below. In this process, the frequency of scattered photon is changed to í´ and the recoiled electron suffers Compton Effect due to gain in the momentum.

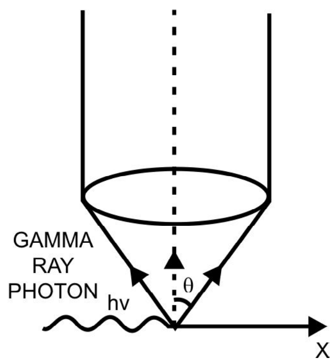

Fig. 1.8

The uncertainty in momentum of the electron can be calculated as follows:

Let a photon of momentum hí/c strike an electron initially at rest. The striking photon transfers a momentum mv to the electron and scatters into microscope as shown in figure below.

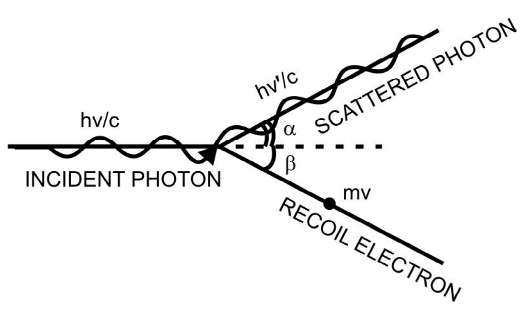

Fig. 1.9

According to principle of conservation of momentum along X-axis, Momentum of Photon before collision +

Momentum of Electron before collision =

Momentum of scattered photon + Momentum of recoil electron

Therefore

$$\frac{hv}{c} + 00 = \frac{hv'}{c}\cos\alpha + mV\cos\beta$$

so that the component of momentum along X-axis transferred by the photon to an electron is given by

$$px = mV\cos\beta = \frac{h\nu}{c} - \frac{h\nu'}{c}\cos\alpha$$

The limits of angle α within the microscope are clearly from $(90-\theta)\,\text{to}\,(90+\theta)$ and so the spread in the X-component of momentum will be given by

$$\frac{h}{c}\{v - v'\cos(90-\theta)\} \le p_x \le \frac{h}{c}\{v - v'\cos(90+\theta)\}$$

$$\frac{h}{c}\{v - v'\sin(\theta)\} \le p_x \le \frac{h}{c}\{v - v'\sin(\theta)\}$$

Therefore the uncertainty in momentum is given by

$$\Delta p_x = \frac{h}{c}\{v + v'\sin(\theta)\} - \frac{h}{c}\{v - v'\sin(\theta)\}$$

$$= \frac{2h\nu'}{c}\sin\theta$$

$$= \frac{2h}{\lambda}\sin\theta \ldots\ldots\ldots\ldots \left(\text{since}\,\lambda = \frac{c}{\nu'}\right)$$

Multiply (1) and (2),

$$\Delta x \Delta p_x \approx \frac{\lambda}{2\sin\theta}\frac{2h\sin\theta}{\lambda} \quad \text{or}$$

$$\Delta x.\Delta p_x \approx h$$

It states that the product of the uncertainty of the X-component of momentum of the electron and the uncertainty in its position along X-axis is of the order of Planck's constant h which is greater than h/2, i.e. in this case, we have

$$\Delta x.\Delta p_x \ge \frac{\hbar}{2}$$

Hence this experiment shows the validity of Heisenberg's uncertainty principle.

SOLVED PROBLEMS

PHOTON

1. A certain spectral line has wavelength 4000A. Calculate the energy of the photon.

Solution: Wavelength of the spectral line = 4000 A = 4000 × 10^{-10}m

$$\text{Energy of photon } E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}}$$

$$= 4.965\times10^{-19}\,J = \frac{4.965\times10^{-19}}{1.6\times10^{-19}} = 3.1eV$$

2. Calculate the frequency and wavelength of a photon whose energy is 75 eV.

Solution: Given: Energy = 75 eV = 75 x 1.6 ×10^{-19} Joules

Energy of photon, $E = h\nu$

$$\text{Therefore } \nu = \frac{E}{h} = \frac{75\times1.6\times10^{-19}}{6.62\times10^{-34}} = 18.12\times10^{15}\,\text{Hz}$$

$$\text{Wavelength } \lambda = \frac{c}{v} = \frac{3\times10^{8}}{18.12\times10^{15}} = 165.5\times10^{-10}\,\text{meter}$$

3. A 10 Kilowatt radio transmitter operates at a frequency of 880 kHz. How many photons per second does it emit?

Solution: Given: Energy of one photon = $h\nu$ = 6.62 × 10^{-34} × 880 × 10^{3} Joules

Power of the transmitter = 10 kWatt = 10000 Joules/sec

$\therefore$ number of photons emitted per second

= 10000/(6.62 × 10^{-34} × 880 × 10^{3})

= 1.716× 10^{31}

4. Calculate the frequency and wavelength of a photon whose energy is 4.965 × 10^{-19} Joules.

Solution: Given: Energy = 4.965 × 10^{-19} Joules

Energy of photon, E = $h\nu$.

Therefore $\nu = \frac{E}{h} = \frac{4.965\times10^{-19}}{6.62\times10^{-34}} = 8.12\times10^{15}$ Hz

Wavelength $\lambda = \frac{c}{v} = \frac{3\times10^{8}}{8.12\times10^{15}} = 4000\times10^{-10}$ meter

5. A certain spectral line has wavelength 6000A. Calculate the energy of the photon.

Solution: Solve as 1. $(\text{Hint}: E = hc/\lambda)$

COMPTON EFFECT

6. An X-ray photon is found to have doubled its wavelength on being scattered by 90°. Find the energy and wavelength of incident photon.

Solution:

When an X-ray photon is Compton scattered through an angle Φ, the increase in wavelength $\Delta\lambda = \frac{h}{m_0c}(1-\cos\phi)$

For $\Phi = 90°, \cos\Phi = 0$

Therefore,

$$\Delta\lambda = \frac{h}{m_0c} = \frac{6.624\times10^{-34}}{9.107\times10^{-31}\times2.998\times10^{8}}$$

$$= 0.0242\times10^{-10}\text{m} = 0.0242\,\text{A}°$$

If λ is the wavelength of incident photon, then wavelength of scattered photon, $\lambda' = \lambda + \Delta\lambda$.

But, given that $\lambda' = \lambda + \Delta\lambda = 2\lambda$

Therefore $\lambda = \Delta\lambda = 0.0242\text{A}°$

Energy of incident photon = $h\nu$

$$= \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{0.0242\times10^{-10}} = \frac{19.89\times10^{-16}}{0.242}\text{J}$$

$$= \frac{19.89\times10^{-16}}{0.0242\times1.6\times10^{-19}}\text{eV} = 0.51\times10^{6}\text{eV} = 0.51\text{MeV}$$

7. X-rays of wavelength 10.0 pm are scattered from a target. (a) Find the wavelength of the X-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered X-rays. (c) Find the maximum kinetic energy of the recoil electrons.

Solution: Wavelength of the incident X-rays = 10.0 pm.

(a) From Compton effect

$$\lambda' - \lambda = \lambda_c(1 - \cos\phi)$$

$$\therefore \quad \lambda' = \lambda + \lambda_c(1 - \cos 45°)$$

$$= 10.0\text{pm} + 0.293\lambda_c$$

$$= 10.7 \text{ pm}$$

(b) $\lambda' - \lambda$ is maximum when $(1 - \cos\phi) = 2$, in which case

$$\lambda' = \lambda + 2\lambda_c = 10.0 \text{ pm} + 4.9 \text{ pm} = 14.9 \text{ pm}$$

(c) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$$\text{K.E.}_{max} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

where λ' is given in (b). Hence

$$\text{K.E.}_{max} = \frac{(6.626 \times 10^{-34}\text{J.s})(3.00 \times 10^{8}\text{m/s})}{10^{-12}\text{m/pm}}\left(\frac{1}{10.0\text{pm}} - \frac{1}{14.9\text{pm}}\right)$$

$= 6.54 \times 10^{-15}$ Joule

DE-BROGLIE WAVE

8. Find the de-Broglie wavelength associated with an electron with a velocity 10^7 m/s.

Solution: According to de Broglie, the wavelength λ associated with any moving particle of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$, where h is Planck's constant

$$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{7}} = 7.3 \times 10^{-11}\text{m}$$

This wavelength is comparable with the atomic dimensions. Hence a moving electron exhibits a wave character.

9. Find the velocity of an electron whose de-Broglie wavelength is 1.2 A°.

Solution: According to de Broglie, the wavelength λ associated with any moving particle of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{where h is Planck's constant}$$

$$\therefore \quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.2 \times 10^{-10}}$$

$$= 0.6 \times 10^{7} \text{ms}$$

10. A ball of mass 10gm has velocity 1.0 m/s. Calculate the wavelength associated with it. Why this wave nature does not show up in our daily observations?

Solution: According to de Broglie, the wavelength λ associated with any moving particle of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \quad \text{where h is Planck's constant}$$

$$= \frac{6.63 \times 10^{-34}}{10 \times 10^{-3} \times 1.0}$$

$$= 6.62 \times 10^{-32} \text{m}$$

The wavelength is much smaller than the dimensions of the ball. Therefore in such cases wave-like properties of matter cannot be observed in our daily observations.

11. Calculate the de-Broglie wavelength associated with a proton moving with a velocity equal to 1/20th of the velocity of light.

Solution: Velocity of proton

$$v = \frac{\text{velocity of light}}{20}$$

$$= \frac{3 \times 10^{8}}{20}$$

$$= 1.5 \times 10^{7} \text{m/s}$$

and mass of proton, m = 1.67×10^{-27} kg

Planck's constant, h = 6.63×10^{-34} Js

Therefore, de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \text{ where h is Planck's constant}$$

$$= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.5 \times 10^{7}}$$

$$= 2.634 \times 10^{-14} \text{m}$$

12. A photon and a deuteron have the same K.E. Which has longer wavelength?

Solution: Let m be the mass of proton and Vp its velocity.

Therefore, Mass of deuteron = 2m. Let Vd be its velocity.

K.E. of proton = ½ mVp^2

K.E. of dueteron = ½ $2mVd^2 = mVd^2$

As the proton and deuteron have the same K.E.

½ $mVp^2 = mVd^2$

$$\therefore \quad V_d = \frac{1}{\sqrt{2}} V_p$$

de-Broglie wavelength of moving proton $\lambda_p = \frac{h}{mV_p}$

de-Broglie wavelength of moving proton $\lambda_d = \frac{h}{2mV_d}$

Therefore $\frac{\lambda_d}{\lambda_p} = \frac{1}{\sqrt{2}}$

$\therefore \quad \lambda_p = \sqrt{2}\lambda_d$ **i.e. proton has a higher de-Broglie wavelength.**

13. Calculate de-Broglie wavelength of an α particle accelerated through a potential difference of 04kV.

Solution: Mass of á particle m = 4 amu = 4 × 1.66 × 10^{-27} kg

= 6.64 × 10^{-27} kg

Charge on the α particle q = 2e = 2 × 1.6 × 10^{-19}

C = 3.2 × 10^{-19} C

We know, de Broglie wavelength for accelerated charged particle–

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Here V= 04 kV = 4000 Volts

Therefore $\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-10} \times 4000}}$

$= 0.16 \times 10^{-12}$ m

14. Calculate de-Broglie wavelength of an electron whose K.E. is 500 eV.

Solution: de-Broglie Wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 500 \times 1.6 \times 10^{-19}}}$$

$$= 0.55 \times 10^{-10} \text{m}$$

15. Calculate the wavelength of thermal neutrons at 27°C.

Given-mass of neutron = 1.67×10^{-27} kg, Plancks constant = 6.63×10^{-34} Js and Boltzmann's constant k = 1.376×10^{-23} JK^{-1}

Solution: de-Broglie Wavelength $\lambda = \frac{h}{\sqrt{3mkT}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.67 \times 10^{-27} \times 1.376 \times 10^{-23} \times 300}}$$

$$= 1.452 \times 10^{-10} \text{m}$$

PHASE VELOCITY & GROUP VELOCITY

16. Find phase and group velocity of an electron whose de-Broglie wavelength (non-relativistic) is 1.2 A°.

Solution: Group velocity Vg = V the particle velocity

Now $= \frac{h}{p} = \frac{h}{\text{mass} \times \text{Particle Velocity}}$

Therefore Velocity of the moving particle

$$\therefore \quad V = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.2 \times 10^{-10}}$$

$$= 6.06 \times 10^{6} \text{m/s}$$

Therefore Group velocity Vg = 6.06 x 10^6 m/s

we know, the phase velocity

$$v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p} = \frac{\hbar\omega}{\hbar k} = \frac{\omega}{k}$$

$$V_p = \frac{E}{p} = \frac{p^2/2m}{p} = \frac{p}{2m} = \frac{h}{2m\lambda} = \frac{6.63 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 1.2 \times 10^{-10}}$$

$= 3.03 \times 10^6$ m/s

From this, we find that phase velocity is one half of the group velocity or particle velocity.

HEISENBERG UNCERTAINTY PRINCIPLE

17. A microscope, using photons, is employed to locate an electron in an atom within a distance of 0.2 A°. What is the uncertainty in the momentum of the electron located in this way?

Solution: Here, Δx = 0.2 A° = 0.2 × 10^{-10} m and Δp = ?

According to uncertainty relation, $\Delta x.\Delta p \approx \hbar$

$$\therefore \quad \Delta p = \frac{\hbar}{\Delta x} = \frac{1.05 \times 10^{-34}}{0.2 \times 10^{-10}}$$

$$= 5.274 \times 10^{-24} \text{ joule.sec or kg.m.s}^{-1}$$

$$= \frac{5.274 \times 10^{-24}}{1.6 \times 10^{-19}}$$

$$eV = 3.36 \times 10^{-5} eV$$

Hence the uncertainty in momentum

= 5.274 × 10^{-24} joule.sec.

18. An electron is confined to a box of length 1.1 × 10^{-8} **meter. Calculate the minimum uncertainty in its velocity, Given m_e = 9.1 × 10^{-1} kg, h = 1.05 × 10^{-34} joule-sec.**

Solution: According to uncertainty relation,

$\Delta x.\Delta p \approx \hbar$ ....***

so that if Δx is maximum, Δp must be minimum, i.e.

$$(\Delta x)_{max}.(\Delta p)_{min} = \hbar,$$

Given: $(\Delta x)_{max}$ = minimum uncertainty in position

$= 1.1 \times 10^{-8}$ meter and

$h = 1.05 \times 10^{-34}$ joule-sec

So that, we have $$(\Delta p)_{min} = \frac{\hbar}{(\Delta x)_{max}} = \frac{1.05 \times 10^{-34}}{1.1 \times 10^{-8}} \text{kg.m/s}$$

$$= 9.1 \times 10^{-27} \text{kg.m/s}$$

But $(\Delta p)_{min} = m.(\Delta V)_{min}$

So that, we have

$$(\Delta V)_{min} = \frac{(\Delta p)_{min}}{m} = \frac{9.1 \times 10^{-27}}{9.1 \times 10^{-31}} \text{m/s}$$

$$= 1.0 \times 10^{4} \text{m/s}$$

19. Find the uncertainty in the momentum of a particle when its position is determined within 0.01 cm.

Given h = 1.05 × 10^{-34} joule-sec.

Solution: According to Heisenberg's uncertainty relation,

$$\Delta x.\Delta p \approx \hbar \quad ....***$$

$$\therefore \quad \Delta p \approx \frac{\hbar}{\Delta x}$$

Here h = 1.05 × 10^{-34} joule-sec and $\Delta p = 0.01 \text{ x } 10^{-2}$ m

$$\therefore \quad \Delta p \approx \frac{1.05 \times 10^{-34}}{0.01 \times 10^{-2}}$$

$$= 1.05 \times 10^{-30} \text{ kg.m/sec.}$$

Hence, the uncertainty in momentum of particle

= 1.05 × 10^{-30} kg.m/s.

20. An electron has a speed 1.05 × 10^{4} m/s within the accuracy of 0.01%. Calculate the uncertainty in the position of the electron, Given m_e = 9.1 × 10^{-31} kg, h = 1.05 x 10^{-34} joule-sec.

Solution: Given m = 9.1 × 10^{-31} kg, h = 1.05 × 10^{-34} joule-sec. and the uncertainty in velocity = 0.01/100 × 1.05 x 10^{4} m/s (because 0.01% = 0.01/100)

Therefore, $(\Delta p) = m.(\Delta V) = 9.1 \times 10^{-31} \times \frac{0.01}{100} \times 1.05 \times 10^{4} \text{kg.m/s}$

But, according to uncertainty relation $\Delta x.\Delta p \approx \hbar$ ....***
Therefore,

$$\Delta x = \frac{\hbar}{\Delta p} = \frac{1.05 \times 10^{-34}}{9.1 \times 10^{-31} \times \frac{0.01}{100} \times 1.05 \times 10^{4}}$$

$$= 1.1 \times 10^{-4} \text{meter}$$

Hence, the uncertainty in position of electron = 1.1 x 10^{-4} m.

21. A nucleon is confined to nucleus of radius 5 × 10^{-15} meters. Calculate the minimum uncertainty in the momentum of the nucleon. Also calculate the minimum K.E. of the nucleon. Given m = 1.67 × 10^{-27} kg, h = 1.05 × 10^{-34} joule-sec.

Solution: According to uncertainty relation,

$$\Delta x.\Delta p \approx \hbar \quad ....***$$

i.e. We know, $(\Delta x)_{max}.(\Delta p)_{min} = \hbar$,

Given: maximum uncertainty in position Δx = diameter = 2 × 5 × 10^{-15} meters and h = 1.05 × 10^{-34} joule-sec.

$$\therefore \quad (\Delta p)_{min} = \frac{\hbar}{(\Delta x)_{max}} = \frac{1.05 \times 10^{-34}}{2 \times 5 \times 10^{-15}}$$

$$= 1.05 \times 10^{-20} \text{kg.m/s}$$

Now, since p cannot be less than $(\Delta p)_{min}$, so we have $p_{min} = (\Delta p)_{min}$

$$\therefore \quad E_{min} = \frac{p^2_{min}}{2m},$$

$$= \frac{(1.05 \times 10^{-20})^2}{2 \times 1.67 \times 10^{-27}}$$

$$= 3.3 \times 10^{-14} \text{joule}$$

22. The average time that an atom retains excitation energy (i.e. life time) is 10^{-8} sec. Calculate the minimum uncertainty with which the excitation energy of the emitted radiation can be determined.

Solution: According to uncertainty relation,

$$\Delta E.\Delta t \approx \hbar \quad \text{..........***}$$

Where ΔE is the uncertainty in energy and Δt is the uncertainty in time.

Given: Δt = 10-8 sec.

$$\therefore \quad \Delta E = \frac{\hbar}{\Delta t} = \frac{1.05 \times 10^{-34}}{10^{-8}}$$

$$= 1.05 \times 10^{-26} \text{ joule.sec}$$

$$= \frac{1.05 \times 10^{-26}}{1.6 \times 10^{-19}}$$

$$eV = 6.56 \times 10^{-8} \text{ eV}$$

Therefore, uncertainty in energy = 6.56× 10^{-8} eV. This is known as energy width of an excited state.